Math 4030　Assignment 2 (Problems)

Due: **Sept**. **28**, **4:00PM**

In this assignment, you will be practicing on the followings.

● Random experiment, outcome, sample space, and events

● Union, intersection, and complement of event

● Mutually exclusive events

● List all outcomes in a sample space

● Count the number of outcomes in an event

● Use of tree diagram

● Rules of multiplication, permutation, factorial, and combination

● Axioms of probability (of events)

● Classical probability calculation

● Conditional probability

● Independent events

● Bayes' Theorem.

1. Suppose that a balanced die (with six sides numbered 1, 2, 3, 4, 5, and 6) is rolled twice.

  a. List all outcomes in the sample space.

  b. Let A be the event that first roll is 4, B be the event that the sum of two rolls is 6. List outcomes in $A, B,$ and $A \cap B$, respectively.

  c. Find the probability of $A \cup B$?

  d. Are A and B mutually exclusive?

  e. Are A and B independent?

2. If $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 5\}$, $C = \{1, 2, 4\}$, and $D = \{3, 5\}$, find the following. (Assuming the sample space $S = \{1, 2, 3, 4, 5\}$.)

  a. $A \cap C$

  b. $A \cap B \cap C$

c. $A \cap \bar{C}$

d. $\overline{C \cap D}$

e. $\bar{A} \cup B$

f. $\overline{\bar{A} \cup \bar{B}}$

3. A bag contains 9 red marbles, 6 white marbles, and 10 blue marbles. You draw 3 marbles out at random.

   a. What is the probability that all three marbles are red?

   b. What is the probability that exactly two of the marbles are red?

   c. What is the probability that none of the marbles are red?

4. A laptop completes a certain job in 3 or 4 seconds. When the battery is not low, the task takes 3 seconds 95% of the time and when the battery is low it takes 3 seconds 80% of the time. The battery is low about 10% of the time.

   a. What is the chance the next time the job is run it takes 3 seconds?

   b. If the job took 3 seconds to run just now, what is the chance the battery is low?

5. A committee of size 8 is selected at random from 12 professional people and 28 non-professional people. What is the probability that exactly 3 members of the committee are professional people?

6. The owner of an appliance store is interested in the relationship between the price at which an item is sold (regular or sale price) and the customer's decision on whether to purchase an extended warranty. After analyzing her records, she produced the following joint probabilities (two-way table):

	Purchased extended warranty	Did not purchased extended warranty
Regular Price	0.21	0.56
Sale Price	0.1	0.13

   a. What is the probability that a customer who bought an item at the

regular price purchases the extended warranty?

b. What is the probability that a customer buys an extended warranty?

7. A carton of 20 rechargeable batteries contains 8 that are defective. In how many ways can the inspector choose 5 of the batteries and get 3 defective ones?

8. Calculate the reliability (to three decimal places) of the system described in the following figure. The probabilities of failure for each component is given. Note that the components work independently of one another.

$$P(A\,fails) = 0.1$$
$$A$$

$$C$$
$$P(C\,fails) = 0.4$$

$$B$$
$$P(B\,fails) = 0.1$$
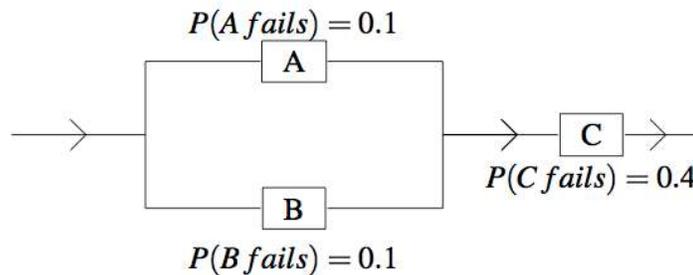
Find the reliability of the system, i.e. the probability that the system will not fail.

9. Consider a sample space of three outcomes $A, B,$ and C. Which of the following represent legitimate probability models? (Hint: check all the axioms.)

A. $P(A) = 0.1, P(B) = 0, P(C) = 0.8$

B. $P(A) = -0.8, P(B) = 1, P(C) = 0.8$

C. $P(A) = 0.1, P(B) = 0.6, P(C) = 0.3$

D. $P(A) = 0.6, P(B) = 0.4, P(C) = 0.2$

E. $P(A) = 0.5, P(B) = 0.5, P(C) = 0.$

10. Three airlines serve a small town in Ohio. Airline A has 46% of all scheduled flights, airline B has 30% and airline C has the remaining 24%. Their on-time rates are 81%, 64%, and 45%, respectively. A flight just left on-time. What is the probability that it was a flight of airline A?